

BER is decreases with
number of Rx antennas.

For L antennas,

$$\text{approx BER} = 2^{L-1} C_L \cdot \left(\frac{1}{2 \text{SNR}} \right)^L$$

For $L = 1$,

$$\begin{aligned} \text{BER} &= {}^1C_1 \cdot \left(\frac{1}{2 \text{SNR}} \right)^1 \\ &= \frac{1}{2 \cdot \text{SNR}} \propto \frac{1}{\text{SNR}} \end{aligned}$$

For $L = 2$

$$\begin{aligned} \text{BER} &= {}^3C_2 \cdot \frac{1}{2^2} \cdot \frac{1}{\text{SNR}^2} = \frac{3}{4} \frac{1}{\text{SNR}^2} \\ &\propto \frac{1}{\text{SNR}^2} \end{aligned}$$

For $L=3$ Antennas,

$$BER = {}^5C_3 \cdot \left(\frac{1}{2 \text{ SNR}} \right)^3$$

$$= \frac{5!}{3! 2!} \cdot \frac{1}{8} \cdot \frac{1}{\text{SNR}^3}$$

$$= \frac{5}{4} \cdot \frac{1}{\text{SNR}^3} \propto \frac{1}{\text{SNR}^3}$$

In General with
 L antennas,

$$\text{BER} \propto \frac{1}{\text{SNR}^L}$$

why is this
arising?

Deep Fade Analysis for multiple Antennas:

$$\bar{y} = \bar{h} x + \bar{n}$$
$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_L \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_L \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_L \end{bmatrix}$$

SNR after Maximal
Ratio Combining or MRC

$$\text{SNR}_m = \frac{P \cdot \|h\|^2}{\sigma^2}$$

$$g = \|h\|^2$$

→ $\frac{gP}{\sigma^2}$

$$SNR = \frac{gP}{\sigma^2}$$

Received power = gP

Noise power = σ^2

Deep fade occurs, when
Received Power < Noise Power

$$\Rightarrow gP < \sigma^2$$

$$\Rightarrow g < \frac{\sigma^2}{P} = \frac{1}{\text{SNR}}$$

$$\boxed{g < \frac{1}{\text{SNR}}}$$

Condition for
Deep Fade

$$\Pr(\text{Deep Fade}) \xrightarrow{g = \|h\|^2} = \Pr\left(g < \frac{1}{\text{SNR}}\right)$$

χ^2 random variable
 $2L$ degrees of freedom

Probability Density
 Function

$$F_G(g) = \frac{g^{L-1}}{(L-1)!} e^{-g}$$

$$\begin{aligned}
 & \Pr\left(g < \frac{1}{\text{SNR}}\right) \quad \leftarrow \text{condition for deep fade} \\
 &= \int_0^{\frac{1}{\text{SNR}}} F_G(g) dg \\
 &= \int_0^{\frac{1}{\text{SNR}}} \frac{g^{L-1}}{(L-1)!} e^{-g} dg
 \end{aligned}$$

At high SNR,

$$\frac{1}{\text{SNR}} \approx 0$$

$$0 \leq g \leq \frac{1}{\text{SNR}}$$

$$e^{-g} \approx 1$$

$$\frac{g^{L-1}}{(L-1)!} e^{-g} \approx \frac{g^{L-1}}{(L-1)!}$$

$\Pr(\text{Deep Fade})$

$$= \int_0^{\frac{1}{\text{SNR}}} \frac{g^{L-1}}{(L-1)!} dg$$

$$= \frac{1}{(L-1)!} \left. \frac{g^L}{L} \right|_0^{\frac{1}{\text{SNR}}}$$

$$= \frac{1}{L!} \frac{1}{\text{SNR}^L} = \frac{1}{L!} \frac{1}{\text{SNR}^L}$$

Probability of Deep Fade

$$P_{DF} = \frac{1}{L!} \cdot \frac{1}{\text{SNR}^L}$$

$$\rightarrow \propto \frac{1}{\text{SNR}^L}$$

$$P_{DF} \propto \frac{1}{\text{SNR}^L}$$

number of
antennas.

With increasing number
of antennas L , Probability
of deep Fade decreases.

This $\uparrow$ leads to a significant
decrease in BER.